1. A one-dimensional flow is described by the velocity field

$$u = ay + by^2 \qquad v = w = 0$$

where a and b are constants. Is the flow irrotational? For what combination of constants, (if any) will the rate of angular deformation given by

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

be zero?

2. The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$$u = 2xy \qquad v = x^2 - y^2$$

Show that the flow is irrotational and satisfies conservation of mass.

3. Definition of stream function ψ from velocity field is

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$$

By substituting these relations in the continuity equation, what will be obtained?

Then according to above relations, for the velocity component of an incompressible two-dimensional velocity field

$$u = x^2 \qquad v = -2xy + x$$

determine, if possible, the corresponding stream function.

4. In a two-dimensional, incompressible flow field, the x component of velocity field is given by the equation $u = 2x$. (a) Determine the corresponding equation for the y component of velocity if $v = 0$ along the x axis. (b) For this flow field, what is the magnitude of the average velocity of the fluid crossing the surface OA of Fig. 1? Assume that the velocities are in feet per second when x and y are in feet.

5. The stream function for an incompressible flow field is given by the equation

$$\psi = 3x^2 y - y^3$$

where the stream function has the units of m^2/s with x and y in meters. (a) Sketch the streamline(s) passing through the origin. (b) Determine the rate of flow across the straight path AB shown in Fig. 2.

6. Make use of the control volume shown in Fig. 3 to derive the continuity equation in cylindrical coordinate

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

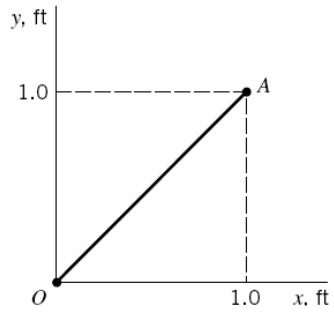

Fig. 1

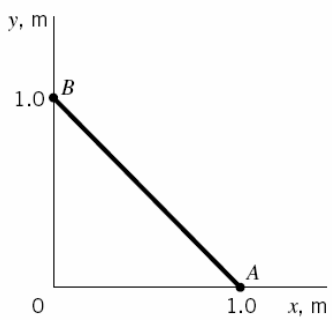

Fig. 2

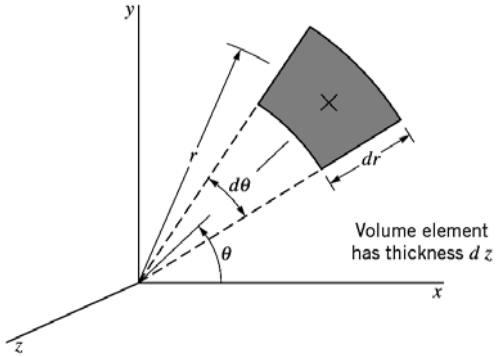

Fig. 3

7. (Computer program) The stream function for an incompressible two-dimensional flow field is

$$\psi = 3x^2 y + y$$

For this flow fields plot several streamlines.